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THE ACTION OF AN UNSTEADY PRESSURE PULSE ON A BODY IN A LIQUID OR GAS

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THE ACTION OF AN UNSTEADY PRESSURE PULSE ON A BODY IN A LIQUID OR GAS

A. I. Golubinskiy, M. N. Kogan

ABSTRACT. Demonstrated is the use of linear theory to avoid making the detailed computations required to obtain the nonstationary flow picture by reducing the determination of the pulses to a problem with fewer measurements.

The difficulties involved in solving differential equations increase sharply /113* with increase in the number of independent variables. Hence, the solution for nonstationary problems is much more complicated than it is for the corresponding stationary problems.

At the same time, knowledge sometimes can be limited to summed pulses, pressures, forces, or moments for a period, or for the entire time the nonstationary process is taking place, and not to each moment in time, during the strength computation, or during the dynamics computation.

This paper demonstrates that, within the limits of linear theory, a detailed computation of the nonstationary flow picture can be avoided by reducing the determination of the pulses to a problem with fewer measurements.

The idea of reducing the dimensionality of the problem is analogous to the idea in [1], being at the same time the known generalization of the approach used in classical problems of pulse motion, or of impact on a fluid [2, 3].

1. Let us consider a flow with undisturbed parameters (U, 0, 0), ρ_∞, p_∞ at infinity around a body, or group of bodies. It is assumed that the angle of inclination of the surface of the body to the flow, and the rate of transfer or deformation of the surface is low, such that the flow can be considered only slightly disturbed and satisfying the linearized gas-dynamics equations

$$\frac{\partial \rho}{\partial t} + U\frac{\partial \rho}{\partial x} + \rho_\infty\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

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* Numbers in the margin indicate pagination in the foreign text.

$$\frac{\partial u}{\partial t}+U\frac{\partial u}{\partial x}+\frac{1}{\rho_\infty}\frac{\partial p}{\partial x}=0,\quad \frac{\partial v}{\partial t}+U\frac{\partial v}{\partial x}+\frac{1}{\rho_\infty}\frac{\partial p}{\partial y}=0$$
$$\frac{\partial w}{\partial t}+U\frac{\partial w}{\partial x}+\frac{1}{\rho_\infty}\frac{\partial p}{\partial z}=0 \qquad (1.1)$$
$$\frac{\partial}{\partial t}(p-a_\infty^2\rho)+U\frac{\partial}{\partial x}(p-a_\infty\rho)=0\quad\left(a_\infty^2=\frac{\varkappa p_\infty}{\rho_\infty}\right)$$

Here, u, v, w are the disturbances for the velocity components, p and ρ are the pressure and density disturbances. The Mach number is sub- or supersonic.

Let us use S to designate the fixed surface of the body, or a surface close to it, at which the boundary conditions at the body can be swept away at any moment in time, t, with an accuracy to within the squares of the disturbing magnitudes. In the case of flow around a group of bodies, surface S is multiply connected.

In the general case, the condition for no flow at S is in the form

$$a(s)v+b(s)w=F(t,\ s) \qquad (1.2)$$

where

s is a point on surface S;

a, b, and F are assigned functions.

The function F(t, s) can be determined by a specified law of motion or deformation of the body, or by the disturbances created by a progressive gust, or wave. In the latter case the disturbed motion of the gas can be given in the form of the sum /114

$$\Gamma=\Gamma_1(t,\ x)+\Gamma_2(t,\ x) \qquad (1.3)$$

Here $\Gamma\equiv$ (u, v, w, ρ, p) is used for purposes of brevity and replaces the set of hydrodynamic magnitudes indicated in the parentheses, and the subscripts 1 and 2 refer, respectively, to the progressive gust, and to the flow it has disturbed. In this problem the function

$$F(t,\ s)=-[a(s)v_1+b(s)w_1]$$

is determined by the specified parameters for the gust, Γ_1, with Eq. (1.1),

given the boundary condition of Eq. (1.2), serving to determine disturbances Γ_2. And it is assumed that the rate of gust disturbed motion, or deformation of the body, is small compared with gust velocities v_1 and w_1, and can be ignored.

A vortex sheet that, within the limits of linear theory, can be swept over a surface formed by the flow lines of an undisturbed flow passing through the trailing edge, descends from the trailing edges of the feeding sheets. The condition of pressure continuity, $p^+ = p^-$, should be satisfied at the sheet. The Zhukovskiy condition, which, within the limits of the linearized theory is equivalent to the satisfaction of the equality $p^+ = p^-$ at the trailing edge, should be satisfied at the subsonic trailing edges of the feeding sheets.

2. Let the boundary conditions, and the conditions in the forward flow before t = 0 and after t = T be stationary, and identical. That is,

$$F(t, s) = F^\circ(s) \quad \text{when } t < 0,\ t > T \tag{2.1}$$

but there is, in the interval $0 \leq t \leq T$ some nonstationary motion

$$F(t, s) = F^*(t, s) \quad \text{when } 0 \leqslant t < T \tag{2.2}$$

Obviously, the flow will be disturbed for some period of time, as well as after boundary condition disturbances have ceased; that is when $t > T$.

Let us use Γ° to designate the solution for the stationary problem with Eq. (2.1) boundary conditions, and Γ^* the solution for the nonstationary problem with Eq. (2.1) and Eq. (2.2) conditions. The difference between these solutions (the nonstationary part of the disturbance)

$$\Delta\Gamma = \Gamma^* - \Gamma^\circ \tag{2.3}$$

obviously satisfies the system of Eq. (1.1).

Now let us consider the pulse of the nonstationary part of the pressure acting at some point on the body and of the determinable magnitude

$$p_i = \int_0^\infty \Delta p\, dt = \int_0^\infty (p^* - p^\circ)\, dt \tag{2.4}$$

Pulses of nonstationary disturbances for all remaining magnitudes, Γ_i, are

formed similarly.

Let us integrate Eq. (1.1), written for the magnitudes of $\Delta\Gamma$, with respect to time from 0 to ∞. Considering that $\Delta\Gamma = 0$ when $t \leq 0$, and that $\Delta\Gamma \to 0$ when $t \to \infty$, we are persuaded that pulses Γ_i satisfy the equations for a stationary current in the case of the following boundary conditions

$$a(s)v_i + b(s)w_i = \int_0^T (F^* - F^\circ)\,dt = F_i(s) \quad \text{on } S \tag{2.5}$$

$$p_i^+ = p_i^- \quad \text{(on the sheet and for the subsonic trailing edges)} \tag{2.6}$$

Actually, let us integrate the equation for $\Delta\Gamma$ with respect to t from 0 to some sufficiently large $t = T_1$. /115

The equations for Γ_i then will differ from the corresponding stationary equations by the presence of right sides that are linear with respect to $\Delta\Gamma(T_1)$. The solution for these equations can be rendered in the form of the sum $\Gamma_{i1} + \Gamma_{i2}$, where Γ_{i1} satisfies the homogeneous equations and boundary conditions at (2.5) and (2.6), while Γ_{i2} are inhomogeneous equations and zero boundary conditions. Since $\Delta\Gamma(T_1) \to 0$ when $T_1 \to \infty$, then $\Gamma_{i2} \to 0$ as well.

The presence of small discontinuities will not change the arguments because within the limits of linear theory, discontinuities cannot be considered as the limits of continuous flows.

Thus, the following assertion is valid. A pressure disturbance pulse at an arbitrary point of a body, created by time-limited disturbances of boundary conditions, or of conditions in a progressive flow, is equal numerically to the stationary pressure at corresponding points of the body, the shape of which can be determined through Eq. (2.5). The consequence of this assertion is the property of the quasi-stationary nature of force and moment pulses acting on a wing in the case of nonstationary disturbances [4], because the pulse from the nonstationary forces (moments) is numerically equal to the pulse of the forces (moments) computed on the assumption of the quasi-stationary nature of the flow over the wing at each moment in time.

It is readily seen that a similar assertion can be made with respect to any periodic motions, considering disturbance pulses for period T. Integrating

the equations and the boundary conditions with respect to time from 0 to T, we once again obtain equations for the pulses that correspond to some stationary flow. This assertion is easy to make in this case by the application of known methods using Fourier series, for example [5].

3. Let us consider transient type nonstationary motion, as a result of which the initial stationary flow (1) is transposed into another stationary flow (2) with the passage of time. We have, for these motions for the Eq. (1.2) boundary conditions

$$\begin{aligned} F(t, s) &= F_1^\circ(s) && \text{when} \quad t \leqslant 0 \\ F(t, s) &= F^*(t, s) && \text{when} \quad 0 \leqslant t \leqslant T \\ F(t, s) &= F_2^\circ(s) && \text{when} \quad t \geqslant T \end{aligned} \tag{3.1}$$

Of interest here is the determination of the pulse for the nonstationary portion of the pressure disturbance, expressed by the formula

$$p_j(s) = \int_0^\infty [p^*(t, s) - p_2^\circ(s)]\, dt = \int_0^\infty \Delta p\, dt \tag{3.2}$$

where

$p^*(t, s)$ is the nonstationary pressure disturbance;

$p_2^\circ(s)$ is the steady pressure limit when $t \to \infty$.

Introducing similar designations for the components of the velocity, density, and their pulses, writing Eq. (1.1) for the magnitudes of $\Delta\Gamma$, and integrating them with respect to t from 0 to ∞, we come once again to stationary, but inhomogeneous, equations

$$U\frac{\partial \rho_j}{\partial x} + \rho_\infty\left(\frac{\partial u_j}{\partial x} + \frac{\partial v_j}{\partial y} + \frac{\partial w_j}{\partial z}\right) = \rho_1^\circ - \rho_2^\circ$$

$$U\frac{\partial u_j}{\partial x} + \frac{1}{\rho_\infty}\frac{\partial p_j}{\partial x} = u_1^\circ - u_2^\circ, \quad U\frac{\partial v_j}{\partial x} + \frac{1}{\rho_\infty}\frac{\partial p_j}{\partial y} = v_1^\circ - v_2^\circ$$

$$U\frac{\partial w_j}{\partial x} + \frac{1}{\rho_\infty}\frac{\partial p_j}{\partial z} = w_1^\circ - w_2^\circ$$

$$U\frac{\partial}{\partial x}(p_j - a_\infty^2 \rho_j) = (p - a_\infty^2 \rho)_1^\circ - (p - a_\infty \rho)_2^\circ \tag{3.3}$$

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for the boundary condition.

$$a(s)v_j + b(s)w_j = F_j(s) = \int_0^\infty [F^*(t, s) - F_2^\circ(s)]\, dt \tag{3.4}$$

and the conditions at the subsonic trailing edges and vortex sheet

$$p_j^+ - p_j^- = 0 \tag{3.5}$$

Let us make further simplifications for the potential (everywhere with the exception of individual surfaces) flow, which, in this case, can be described by the equation

$$-\frac{1}{a_\infty^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{2M_\infty^2}{U}\frac{\partial^2\varphi}{\partial t\,\partial x} + (1-M_\infty^2)\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0 \tag{3.6}$$

We recall (see 1) that in the case when a gust is encountered the solution is in the form of the sum $\Gamma_1 + \Gamma_2$, where Γ_1 is the solution corresponding to the gust itself, and that here only the solution for Γ_2 is considered. It is apparent that the solution for Γ_2 is vortex-free when gust Γ_1 is vortical. The pressure can be expressed in terms of the potential

$$p^\circ = -\rho_\infty U\varphi_x, \quad p^* = -\rho_\infty(U\varphi_x + \varphi_t) \tag{3.7}$$

We are once again interested in the magnitude

$$p_j = \int_0^\infty (p^* - p_2^\circ)\,dt = -\rho_\infty(U\varphi_{jx} + \varphi_2^\circ - \varphi_1^\circ) \tag{3.8}$$

where

$$\varphi_j = \int_0^\infty (\varphi^* - \varphi_2^\circ)\,dt \tag{3.9}$$

Taking the difference of Eq. (3.6) for φ^* and φ_2°, and integrating that difference with respect to t from 0 to ∞, we obtain

$$(1-M_\infty^2)\frac{\partial^2\varphi_j}{\partial x^2} + \frac{\partial^2\varphi_j}{\partial y^2} + \frac{\partial^2\varphi_j}{\partial z^2} = 2\frac{M_\infty^2}{U}(\varphi_{2x}^\circ - \varphi_{1x}^\circ) \tag{3.10}$$

The boundary condition for φ_j will be

$$a\varphi_{jy} + b\varphi_{jz} = \int_0^\infty (F^* - F_2^\circ)\,dt = F_j(s) \tag{3.11}$$

given the condition at the subsonic trailing edges and at the vortex sheet of

$$v(\varphi_{jx}^+ - \varphi_{jx}^-) = -\varphi_2^\circ + \varphi_2^\circ + \varphi_1^\circ - \varphi_1^\circ \tag{3.12}$$

It is readily seen that the general solution for Eq. (3.10) can be given in the form /117

$$\varphi_j = \varphi^* + \frac{M_\infty^2}{(1-M_\infty^2)U}(\varphi_2^\circ - \varphi_1^\circ)x \tag{3.13}$$

where φ^* satisfies the equation for the steady flow

$$(1-M_\infty^2)\frac{\partial^2\varphi^*}{\partial x^2} + \frac{\partial^2\varphi^*}{\partial y^2} + \frac{\partial^2\varphi^*}{\partial z^2} = 0 \tag{3.14}$$

for the boundary condition at surface S

$$a\varphi_y^* + b\varphi_z^* = F_j(s) - \frac{M_\infty^2 x}{(1-M_\infty^2)U}(F_2^\circ - F_1^\circ) \tag{3.15}$$

and the condition at the subsonic trailing edges and at the vortex sheet

$$\varphi_x^{*+} - \varphi_x^{*-} = -\frac{1}{U(1-M^2)}(\varphi_2^{\circ+} - \varphi_2^{\circ-} - \varphi_1^{\circ+} + \varphi_1^{\circ-}) \tag{3.16}$$

The unknown pulse equals

$$p_j = -\rho_\infty U\varphi^* + \frac{M_\infty^2}{(1-M_\infty^2)U}(p_2^\circ - p_1^\circ)x - \frac{\rho_\infty}{1-M_\infty^2}(\varphi_2^\circ - \varphi_1^\circ) \tag{3.17}$$

We should point out that in the auxiliary stationary problem for φ^*, the condition of Eq. (3.16) differs from the conventional at the subsonic trailing edges and at the sheet, in that velocity $u^* = \varphi_x^*$ suffers a discontinuity equivalent to the specified distribution of the vortices at the sheet. The strength of this additional sheet is proportional to the difference in lift forces under stationary conditions before and after the transient. Its appearance is the result of the summing with respect to time of the vortices trailing back as a result of change in lift force.

Accordingly, the nonstationary problem is reduced to the stationary problem of the flow around a body of Eq. (3.15) when vortices are present at the sheet, the strength of which is determined by the condition of Eq. (3.16). Available, efficient, methods of computing stationary flows can be used for solving this problem, after certain modifications are made.

Let us give some examples of the solution of nonstationary problems, by way of illustration, using the method described.

Example 1. There is a sudden change in the angle of attack of a two-dimensional plate from 0 to 1 in the case of a supersonic flow over the plate. Given for this problem are

$$F(t, s) = 0 \quad \text{when} \quad t \leqslant 0$$
$$F(t, s) = U \quad \text{when} \quad t \geqslant 0$$

The boundary condition for φ^* is in the form

$$\varphi_y^* = \frac{M_\infty^2}{(M_\infty^2 - 1)} x \tag{3.18}$$

The Ackeret formula can be used to compute pressures p_2^0 and φ_x^*. Then, in accordance with Eq. (3.17), we have for the pulse attributable to the nonstationary portion of the pressure on the plate

$$p_j = \mp \rho U x (M^2 - 1)^{-1/2} \tag{3.19}$$

The minus sign is for the upper surface, the plus for the lower.

This result can be obtained, naturally, in a somewhat more complex form from the completely nonstationary solution for the problem (see [5], for example).

Example 2. There is a sudden change in the angle of attack of a delta wing with subsonic leading and straight trailing edges. It is extremely difficult to arrive at a direct solution for this nonstationary problem, and there is no complete solution for it as yet, apparently. The method proposed can compute the pulse for the nonstationary pressures on this wing, or the auxiliary problem for φ^* can be reduced to the problem of the flow around a wing with surface deformation in accordance with a linear law from the x coordinate, the solution for which is known [6]. The computations for potential φ^*, for the corresponding boundary conditions of Eq. (3.15), yield

$$\varphi^* = \frac{N_1(M_\infty) x \sqrt{m^2 x^2 - y^2}}{U}$$

$$N_1 = \frac{M_\infty^2 (1 - m^2)}{(1 - M_\infty^2)\{m[K(m') - E(m')] + (1 - m^2) E(m')\}}$$

$$m = \sqrt{M_\infty^2 - 1}\, \operatorname{tg} \delta, \qquad m' = \sqrt{1 - m^2}$$

where K, E are the perfect elliptic integrals; δ is the half-angle for the wing. The pulse, p_j, on a delta wing with subsonic edges can be computed by using Eq. (3.17)

$$p_j = -\rho_\infty \dot{U}\left(N_1 + \frac{M_\infty^2}{1-M_\infty^2}\right)\left(\sqrt{m^2x^2-y^2} + \frac{m^2x^2}{\sqrt{m^2x^2-y^2}}\right) - \rho_\infty U \frac{\sqrt{m^2x^2-y^2}}{E(m')}.$$

4. In a number of cases the problem allows for a further reduction in dimensionality. For example, let an arbitrary gust (wave), the parameters of which are given by the magnitudes of Γ_1, encounter a body of revolution around which there is a stationary flow with velocity U parallel to the axis of the body. The problem of determining the disturbance in gas motion caused by the gust, Γ_2, is four-dimensional.

In a cylindrical system of coordinates, the x axis of which coincides with the axis of the body, the potential of the disturbed motion, φ_2, satisfies the equation (we shall omit the subscript 2 in what follows)

$$-a_\infty \varphi_{tt} - \frac{2M_\infty^2}{v}\varphi_{tx} + (1-M_\infty^2)\varphi_{xx} + \varphi_{rr} + \frac{1}{r^2}\varphi_{\theta\theta} + \frac{1}{r}\varphi_r = 0 \tag{4.1}$$

at the body

$$\varphi_r = \begin{cases} F_1^\circ(x, r(x), \Theta) & \text{when } t<0 \\ F^*(t, x, r(x), \Theta) & \text{when } 0 \leqslant t \leqslant T \\ F_2^\circ(x, r(x), \Theta) & \text{when } t>T \end{cases} \tag{4.2}$$

$\varphi = 0$ at infinity. Introducing φ_j from Eq. (3.9), and integrating Eq. (4.1) with respect to t from 0 to ∞, we obtain

$$(1-M_\infty^2)\varphi_{jxx} + \varphi_{jrr} + \frac{1}{r^2}\varphi_{j\theta\theta} + \frac{1}{r}\varphi_{jr} = 2\frac{M_\infty^2}{U}(\varphi_2^\circ - \varphi_1^\circ) \tag{4.3}$$

We have the following boundary conditions at the body

$$\varphi_{jr} = \int_0^\infty (F^* - F_2^\circ)\,dt = F_j \tag{4.4}$$

The solution for Eq. (4.3) can be given in the form of Eq. (3.13). Then, for φ^*

$$(1-M_\infty^2)\varphi_{xx}^* + \varphi_{rr}^* + \frac{1}{rr}\varphi_{\theta\theta}^* + \frac{1}{r}\varphi_r^* = 0 \tag{4.5}$$

and the boundary condition is analogous to that of Eq. (3.15).

Thus, the problem has been reduced to a stationary problem of the flow around a body. There is an analogous three-dimensional problem in the general case for the determination of φ_1° and φ_2° as well.

Eq. (3.16) is used to determine the pressure pulse. It is sufficient to know the following three integrals in order to determine the pulses from all forces and moments acting on the body

$$\int_0^{2\pi} p_j\, d\Theta, \quad \int_0^{2\pi} p_j \cos\Theta\, d\Theta, \quad \int_0^{2\pi} p_j \sin\Theta\, d\Theta$$

In accordance with Eq. (3.17), the following integrals must be computed in order to find these integrals /119

$$\varphi^{\circ} = \int_0^{2\pi} \varphi^*\, d\Theta, \quad \varphi^{\cos\Theta} = \int_0^{2\pi} \varphi^* \cos\Theta\, d\Theta, \quad \varphi^{\sin\Theta} = \int_0^{2\pi} \varphi^* \sin\Theta\, d\Theta$$

and analogous integrals of φ_1° and φ_2°. The methods used to find all of them are identical. We shall indicate the manner of their determination only for φ^{Θ}. Integrating Eq. (4.5) with respect to Θ and 0 to 2π, we have

$$(1-M_\infty^2)\varphi_{xx}^{\Theta} + \varphi_{rr}^{\Theta} + r^{-1}\varphi_r^{\Theta} = 0 \tag{4.6}$$

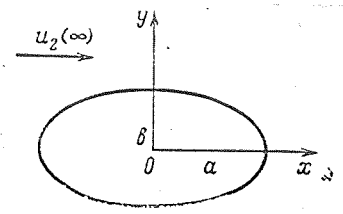
Integrating the corresponding condition for φ^* at the body, we reduce the problem to one of the flow around an axisymmetrical body.

Multiplying Eq. (4.5) by $\cos\Theta$ and $\sin\Theta$, respectively, and integrating, we obtain

$$(1-M_\infty^2)\varphi_{xx}^{\cos\Theta} + \varphi_{rr}^{\cos\Theta} + \frac{1}{r}\varphi_r^{\cos\Theta} = \frac{1}{r^2}\varphi^{\sin\Theta} \tag{4.7}$$

$$(1-M_\infty^2)\varphi_{xx}^{\sin\Theta} + \varphi_{rr}^{\sin\Theta} + \frac{1}{r}\varphi_r^{\sin\Theta} = \frac{1}{r^2}\varphi^{\cos\Theta} \tag{4.8}$$

These equations can be added and subtracted in order to solve the system of Eq. (4.7) simultaneously. We will then obtain independent equations of the same type for $(\varphi^{\cos\Theta} + \varphi^{\sin\Theta})$ and $(\varphi^{\cos\Theta} - \varphi^{\sin\Theta})$. Thus, the determination of $\varphi^{\cos\Theta}$ and $\varphi^{\sin\Theta}$ has been reduced to the axisymmetrical problem that is close to the problem of the axisymmetrical flow around a body of revolution; that is, the four-dimensional problem has been reduced, in the final analysis, to several two-dimensional ones.



And the analogous four-dimensional problem for a wing with supersonic leading edges and a trailing edge that is perpendicular to the flow can be reduced to a problem of the stationary flow around an airfoil. The integration of the corresponding three-dimensional problem for Γ_j with respect to the wing span (see [1]) is sufficient for this purpose.

5. Finally, let us consider the problem of a weak shock wave, or gust, acting on an immovable obstacle. The condition of slenderness of the body can be dispensed with in this problem.[1]

Let a weak shock wave, or gust, specified by the functions of Γ_1, which are the solutions for the linearized gas-dynamics equations, Eq. (1.1), in which U = 0, encounter an arbitrary body in a quiescent gas with parameters ρ_∞, p_∞.

Let us seek the disturbed flow in the form

$$\Gamma = \Gamma_1 + \Gamma_2 \tag{5.1}$$

The solution satisfies Eq. (1.1) and the boundary conditions

$$\left.\begin{array}{l} \Gamma_2 \to 0 \quad \text{when } r \to \infty \\ (q_2 + q_1)n = 0 \text{ at the body} \end{array}\right\} \quad [q = (u,v,w)] \tag{5.2}$$

n is the external normal to S.

Let us take the moment of initiation of the action of the disturbance Γ_1 on the body as t = 0. We then have the following problem for Γ_2. A body, which is in a quiescent gas up to time t = 0 is immovable, but when t > 0 specified disturbances appear on its surface. Flow Γ_2 is vortex-free. This problem differs from that considered in 2 above only in that U = 0. Therefore [see Eq. (3.7)] /120

$$p_2 = -\rho_\infty \varphi_{2t} \tag{5.3}$$

Setting $\varphi_2(t < 0) = 0$, we have

1. The problem of determining the summed pulse acting on a body around which a weak shock wave flows has been considered in [7] in a similar statement of the problem.

$$p_{j1}=\int_0^\infty (p_1-p_1(t=\infty))\,dt$$

$$p_{j2}=\int_0^\infty (p_2-p_2(t=\infty))\,dt=\int_0^\infty p_2\,dt=-\rho_\infty\varphi_2(t=\infty) \qquad (5.4)$$

because the stationary pressure $p_2(t = \infty)$ has a second order of smallness.

Thus, determination of the pressure pulse has been reduced to the computation for a pulse of unstable pressure, p_{j1}, computed as if no body were present, and to the stationary problem of determining $\varphi_2(t = \infty)$.

If the forward gust is finite in extent, obviously $\varphi(\infty) = 0$ and $p_{2i} = 0$. In this case the pressure pulse can be determined completely by the pulse of unstable pressure, p_{il}. Moreover, if the forward gust is one-dimensional, p_{il} will be identical at any point on the body, and, as a result, the summed pulse acting on the closed body will equal zero. Pulse p_{j2} is dependent only on the steady flow limit, and does not depend on the pressure distribution in the gust. The latter can be taken into consideration by pulse p_{j1}, however.

Example. Let us compute the unsteady pressure pulse communicated to an elliptical cylinder by a weak progressive shock wave. From the known expression for the potential of the flow over the surface of an elliptical cylinder without circulation we have (figure)

$$\varphi_2 = u_2(\infty)\frac{b}{a}x$$

from whence, for the pressure pulse p_{j2} using Eq. (5.4), we obtain

$$p_{j2}=\rho_\infty u_2(\infty)\frac{b}{a}x$$

were $u_2(\infty)$ is the air flow velocity behind the shock wave front (perpendicular to the x axis).

The progressive shock wave creates pulse

$$p_{j1}=\rho_\infty u_\infty(\infty)(a+x)$$

The summed x axis component for the total force pulse applied to the cylinder equals

$$p_j=\pi\rho u_2(\infty)b^2(1+a/b)$$

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REFERENCES

1. Kogan, M.N., "Some Properties of Three-Dimensional Supersonic Flows," PMM, 1956, No. 5.

2. Lamb, G., Gidrodinamika [Hydrodynamics], Moscow. Gostekhizdat, 1947.

3. Sedov, L.I., Ploskiye zadachi gidrodinamike i aerodinamiki [Plane Problems of Hydrodynamics and Aerodynamics], Moscow, "Nauka" Press, 1966.

4. Golubinskiy, A.I., "The Pulse of Forces and Moments Communicated to a Thin Wing During Nonstationary Flow Around the Wing," Izv. AN SSSR, MZhG, No. 6, 1968.

5. Miles, J.I. Potentsial'naya teoriya neustanovivshikhsya sverkhzvukovykh techeniy [Potential Theory of Nonstationary Supersonic Flows], Moscow, Fizmatgiz, 1963.

6. Robinson, A., "Rotary Derivatives of a Delta Wing at Supersonic Speeds," Trans. Roy. Aeronaut. Soc., No. 52, 1948.

7. Nikol'skiy, A.A., Smirnov, V.A., "Action of a Shock Wave on an Obstacle," Inzh. zh., No. 1, 1962.

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